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THE METHOD OF SMOOTHING APPLIED TO COMPOSITE ROUGH SURFACE

Virginia Polytechnic Institute and State University

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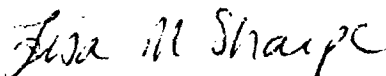


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13. ABSTRACT (Maximum 200 words) A new solution is developed for the problem of scattering from a randomly rough, perfectly conducting, interface having many scales of roughness. A review of the capabilities and limitations of first order smoothing (FOS) and normalized first order smoothing (NFOS) approximations are first carried out. By combining the concepts contained in NFOS approximation with those of the conventional composite surface scattering model, it is possible to develop a new method that is more robust than its components. The key step is to split the regions of current interaction on the rough surface into near and far zones; the effect of the latter is mainly one of large scale surface shadowing while the former accounts for small scale diffraction. Comparisons with the conventional composite surface scattering model are carried out and a number of improvements are noted. Further improvements in the model, if needed, appear to be tractable.				
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TABLE OF CONTENTS

Page

Introduction	1
Background	2
Capabilities and Limitations of Normalized Smoothing	9
A Combination of the Normalized Smoothing and Composite Surface Ideas	12
Discussion of Results	19
Summary	22
References	23

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Introduction

High speed digital computers have had a significant impact on the rough surface scattering community because they provide a new and important problem solving tool. As an example, computer simulations have led to a much better understanding of the root causes of the phenomenon known as enhanced backscattering from rough surfaces [1]. However, to date, it has only been possible to use the full power of the computer on one dimensionally rough surfaces because the two dimensional rough surface scattering problem is still too large to be so solved. There is work being done with techniques which might make the two dimensional problem more tractable on a computer [2]; however, such improvements remain to be thoroughly demonstrated.

This state of affairs leads to the rather obvious conclusion that there is still a need for further analytical work to reduce the numerical difficulty of the problem to a more tractable level. The intent here is to isolate, separate out, and perform as much of the solution as possible using analytical techniques; numerics will only be required for those parts of the problem where analytical approximations are not possible or jeopardize the accuracy of the solution. The key ingredient in such a solution is the development of an approach which is very robust in its capabilities. For example, one would not want to start out with a boundary perturbation type solution in which both the surface height and the surface slopes are required to be small if the solution were going to be applied to surfaces with large slopes. Two approaches have recently been introduced and are being developed which appear to hold the promise of significantly improving our capabilities with regard to predicting the scattering from two dimensional rough surfaces. The first of these is the work of Rodriguez and Kim using what they call the unified perturbation expansion approach [3]; this technique is based on a small momentum transfer approximation developed earlier by Rodriguez [4]. The other approach is based on the method of smoothing [5, 6, 7], a demonstration of its ability to deal with surfaces having arbitrary surface height derivatives for small heights [8], and a possible extension to surfaces having larger heights [9]. It should be noted that both of these approaches hold the promise of extending our prediction capabilities well beyond those of existing asymptotic solutions. The remainder of this report will be concerned with the solutions which are based on the method of smoothing.

Whenever new approximate solutions are obtained for complex problems such as scattering by randomly rough surfaces, it is wise to compare these with existing

approximations. This is done for two reasons. First, it is essential to compare with known benchmarks in order to check the accuracy of the new solution and to see where it provides improvement. A second purpose, which is frequently overlooked, is to generate new ideas as to how the new approximation may be improved even further. It is this latter reason that forms the basis for this report. The basic intent is to compare what has been called the normalized first order smoothing (NFOS) approximation with the conventional composite surface scattering model to (1) first determine what improvements the latter suggests for the former, and (2) to then demonstrate just how important these improvements are. The final result will be a new approximate scattering result which essentially extends the existing composite surface model to a new class of surfaces. Unlike the conventional composite surface model, this new result will also contain a well defined methodology for extending its capabilities in the sense of making it apply to more complicated surfaces. This is a feature that the conventional composite surface model never exhibited.

Before presenting the main topic of this report, it is first necessary to review the first order smoothing (FOS) approximation and its capabilities and limitations. Of particular note is its ability to reduce to the Rice boundary perturbation approximation provided the surface height is very small in terms of the electromagnetic wavelength and the angle of incidence is not too close to grazing. Of equal importance is to show why FOS begins to breakdown as the surface height approaches or exceeds the electromagnetic wavelength. Identification of the failure of FOS as the height becomes large suggests a normalization scheme to overcome this limitation and this leads to the normalized first order smoothing (NFOS) approximation. By analyzing the class of surfaces to which NFOS should apply and comparing these with the surfaces to which the conventional composite surface model applies, a means for improving the NFOS approximation even further is developed. It is this improved NFOS model which extends the basic smoothing technique to a class of surface not heretofore addressable. The final result is a smoothing based scattering approximation that is applicable to the type of multiscale roughness surfaces that are common in nature.

Background

The specific problem to be considered is that of scattering of an incident plane electromagnetic wave by an arbitrarily roughened perfect electric conductor occupying

the half space defined by $z \leq \zeta(x, y)$. The roughness height is taken to be zero mean about the $z=0$ plane, i.e., $\langle \zeta \rangle = 0$, and is assumed to be a statistically homogeneous process. This latter constraint implies that the surface statistics are independent of the point (x, y) where they are being measured. The region above the roughened conducting half space, e.g., $z > \zeta(x, y)$, is taken to be free space. The incident plane wave is traveling in the direction specified by the unit vector \hat{k}_i and \hat{k}_s indicates the direction of scattering, see Figure 1.

The current induced on the conducting surface $\vec{J}_s(\vec{r})$ obeys the magnetic field integral equation, e.g.,

$$\vec{J}_s(\vec{r}) = \vec{J}_s^i(\vec{r}) + 2\hat{n}(\vec{r}) \times \int \frac{\partial g(|\Delta\vec{r}|)}{\partial |\Delta\vec{r}|} \Delta\vec{r} \times \vec{J}_s(\vec{r}_o) dS_o \quad (1)$$

which, after multiplying by

$$\sqrt{1 + (\nabla \zeta)^2} \exp(jk_{sz} \zeta),$$

becomes

$$\vec{J}(\vec{r}) = \vec{J}^i(\vec{r}) + L\Gamma(\vec{r}, \vec{r}_o) \bullet \vec{J}(\vec{r}_o) \quad (2)$$

where

$$\nabla \zeta = \frac{\partial \zeta}{\partial x} \hat{x} + \frac{\partial \zeta}{\partial y} \hat{y} + \hat{z} \quad (3a)$$

$$k_{sz} = k_o \hat{k}_s \bullet \hat{z} \quad (3b)$$

$$\hat{n}(\vec{r}) = (\frac{\partial \zeta}{\partial x} \hat{x} - \frac{\partial \zeta}{\partial y} \hat{y} + \hat{z}) / \sqrt{1 + (\nabla \zeta)^2} = \sqrt{1 + (\nabla \zeta)^2} \vec{N}(\vec{r}) \quad (3c)$$

$$g(|\Delta\vec{r}|) = \exp(-jk_o|\Delta\vec{r}|) / 4\pi|\Delta\vec{r}| \quad (3d)$$

$$\Delta\vec{r} = (\vec{r} - \vec{r}_o) / |\vec{r} - \vec{r}_o| \quad (3e)$$

and $k_o = 2\pi/\lambda_o$ and

$$\vec{J}(\vec{r}) = \vec{J}_s(\vec{r}) \sqrt{1 + (\nabla \zeta)^2} \exp(jk_{sz}\zeta) \quad (4a)$$

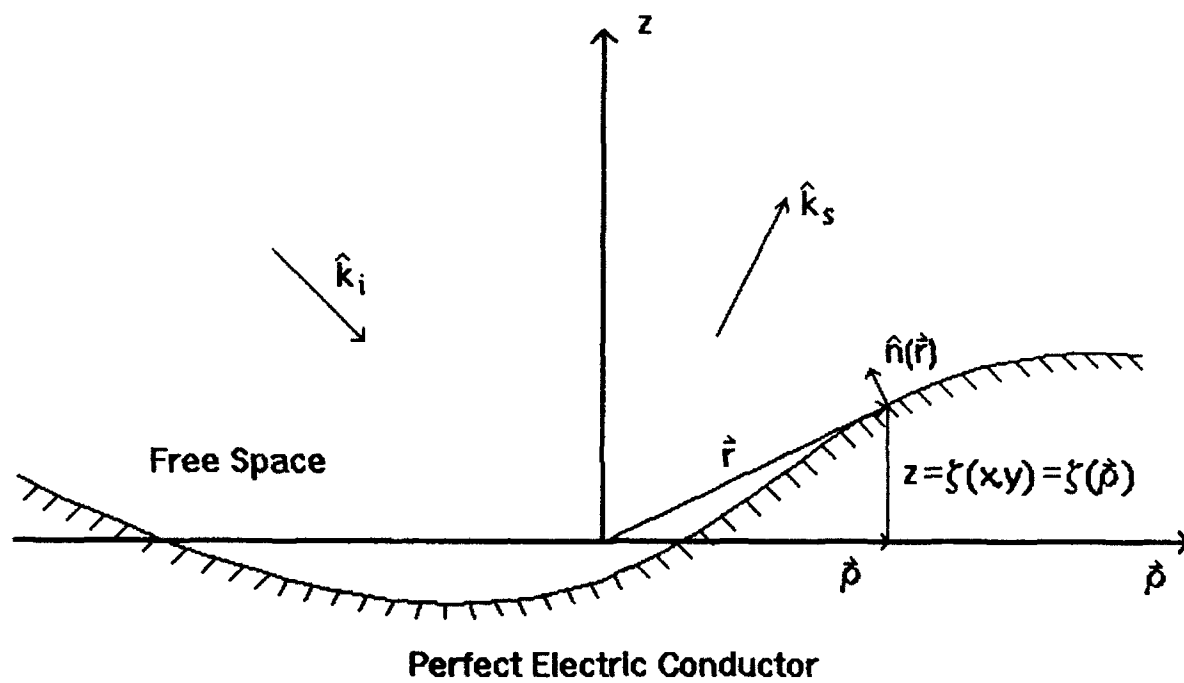


Figure 1. Scattering geometry

$$\vec{J}^i(\vec{r}) = 2\vec{N}(\vec{r}) \times \vec{H}^i(\vec{r}) \exp(jk_{sz}\Delta\zeta) \quad (4b)$$

$$L = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_o dy_o = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\vec{\rho}_o \quad (4c)$$

$$\Gamma = -2 \frac{\partial g}{\partial |\Delta\vec{r}|} \exp(jk_{sz}\Delta\zeta) \vec{N}(\vec{r}) \times [\hat{\Delta}\vec{r} \times \quad (4d)$$

$$\Delta\zeta = \zeta(\vec{\rho}) - \zeta_o(\vec{\rho}_o) \quad (4e)$$

In the above, $\hat{n}(\vec{r})$ is the unit normal to the surface at the point $\vec{r} = \vec{\rho} + \zeta\hat{z}$ on the surface, $\partial\zeta/\partial x$ and $\partial\zeta/\partial y$ are the x and y surface slopes, g is the free space Green's function evaluated on the surface, and \vec{H}^i is the incident magnetic field. It should also be noted that $\Gamma(\vec{r}, \vec{r}_o)$ is a 3x3 matrix operator. The reason behind manipulating (1) into (2) is that a simple Fourier transform of the currents appearing in (2) essentially yields the far zone fields scattered by the rough surface. In fact, previous papers dealing with the smoothing method [8, 9] have found it advantageous to deal with the transform of (2); however, in this paper attention will be directed toward the current equation. The reasons for this will become obvious later.

The basic idea behind smoothing is relatively straightforward. First, the current is split into a zero mean and a fluctuating parts, i.e.

$$\vec{J} = \langle \vec{J} \rangle + \delta\vec{J} \quad (5)$$

Quite apart from the approximations this leads to, (5) is a natural decomposition in that $\langle \vec{J} \rangle$ gives rise to the specular or coherent scattered field while $\delta\vec{J}$ leads to the diffuse or incoherent scattered power. Substituting (5) into (2) and successively taking the average and fluctuating part of the resulting equation generates two coupled integral equations for $\langle \vec{J} \rangle$ and $\delta\vec{J}$. For example, the equation for $\delta\vec{J}$ is

$$\delta\vec{J} = [1 - L\langle\Gamma\rangle]^{-1} \{ \delta\Gamma \bullet \langle \vec{J} \rangle + \delta[L\delta\Gamma \bullet \delta\vec{J}] \} \quad (6)$$

In (6), $\delta\Gamma$ is the fluctuating part of the propagator in (2), and $[1 - L\langle\Gamma\rangle]^{-1}$ is an operator in coordinate space but becomes a multiplicative function in transform space. The specularity of the average scattered field, which has been demonstrated quite independent of the smoothing method [10], implies the following simplification

$$\langle \tilde{J} \rangle = \tilde{J} \exp(-j \tilde{k}_{it} \cdot \tilde{\rho}) \quad (7)$$

where $\tilde{k}_{it} = \tilde{k}_i - k_{iz} \hat{z}$ and \tilde{J} is a vector number which is independent of any of the surface coordinates. Substituting (7) into (6) yields

$$\begin{aligned} \delta \tilde{J} = [1 - L(\Gamma)]^{-1} \{ \delta \tilde{J}^i + \delta \tilde{\Gamma}(\tilde{r}, -\tilde{k}_{it}) \cdot \tilde{J} \\ + \delta [L \delta \Gamma \cdot \delta \tilde{J}] \} \end{aligned} \quad (8)$$

where the tilde over $\delta \Gamma$ denotes the Fourier transform from $\tilde{\rho}_o$ space to \tilde{k}_{it} space.

For most surfaces of interest, a good approximation for \tilde{J} is \tilde{J}^i which is defined by

$$\tilde{J}^i = 2H_o \langle \tilde{N} x \hat{h}_i \exp[j(k_{sz} - k_{iz})\zeta] \rangle \quad (9)$$

where H_o is the amplitude of the incident magnetic field and \hat{h}_i is its vector direction. For Gaussian surfaces, the height and slopes at a common point on the surface are uncorrelated so (9) becomes

$$\tilde{J}^i = 2H_o \langle \tilde{N} x \hat{h}_i \rangle \langle \exp[j(k_{sz} - k_{iz})\zeta] \rangle \quad (10)$$

The average of the exponential height term is recognized to be the characteristic function of the surface height. Substituting (10) in (8), ignoring the operator $[1 - L(\Gamma)]$ since it has already been assumed to be small (in replacing \tilde{J} by \tilde{J}^i), replacing $\delta \Gamma$ by Γ , and retaining only the new Born term in the resulting integral equation yields the first order smoothing (FOS) approximation;

$$\delta \tilde{J}^{(1)} = \delta \tilde{J}^i + 2H_o \langle \exp[j(k_{sz} - k_{iz})\zeta] \rangle \tilde{\Gamma} \cdot \langle \tilde{N} x \hat{h}^i \rangle \quad (11)$$

It can be shown that $\delta \tilde{J}^i$ is the term that is most important about the specular direction, i.e.

$$\hat{k}_s^{(s)} = \frac{\tilde{k}_{it}}{|\tilde{k}_{it}|} - \frac{k_{iz}}{k_o} \hat{z} \quad (12)$$

while the second term is the major contributor far away from the specular direction. In addition, it can be shown (although not easily) that (11) leads to the classic Rice boundary perturbation result provided that $k_o \zeta \ll 1$ and the incidence angle is not too close to grazing incidence. The first restriction permits ignoring the height dependence in $\delta \tilde{\Gamma}$ while the second insures that the terms which are ignored are indeed small relative to the ones retained in the simplification process. It is interesting to point out that simplifying (11) to the Rice limit does not require small slopes; this would tend to indicate that the Rice limit is probably more robust than classical derivations would tend to indicate. Certainly, there are cases in the literature where the Rice limit appears to "work" when the classical restrictions would suggest otherwise [11].

Interestingly enough, the FOS approximation in (11) clearly shows why it fails as the surface height becomes large relative to the EM wavelength. More specifically, as the surface roughness height becomes large the characteristic function for the height in (11) decreases; for a Gaussian surface, the decrease is most rapid. This behavior is clearly incorrect because the increase in height can be brought about by the addition of a randomly elevated planar component to the surface roughness which has no horizontal structure and yet it leads to an attenuation of the wide angle scattering contribution in (11). The difficulty here, as identified and remedied in [9], is that smoothing is only applicable to surfaces having a relatively small height. The remedy for this situation is to return to the original current integral equation, (2), and normalize this equation by the height dependent phase factor appearing in the term \tilde{J}^i in (2). This normalization leads to

$$\tilde{J}_n = \tilde{J} \exp[-j(k_{sz} - k_{iz})\zeta] \quad (13a)$$

$$\tilde{J}_n^i = 2\tilde{N} \times \tilde{H}^i \exp[jk_{iz}\zeta] \quad (13b)$$

$$\Gamma_n = \Gamma(\vec{r}, \vec{r}_o; k_{sz} \rightarrow k_{iz}) \quad (13c)$$

and

$$\tilde{J}_n = \tilde{J}_n^i + L \Gamma_n \bullet \tilde{J}_n \quad (14)$$

Applying smoothing to this normalized equation yields the following integral equation for the fluctuating normalized current;

$$\delta \vec{J}_n = [1 - L(\Gamma_n)]^{-1} \{ \delta \vec{J}_n^i + \delta \tilde{\Gamma}_n \bullet \vec{J}_n + \delta [L \Gamma_n \bullet \delta \vec{J}_n] \} \quad (15)$$

Assuming that \vec{J}_n^i is a good approximation for \vec{J}_n , retaining only the resulting Born term in (15) as the first order approximate solution, and ignoring the operation $[1 - \langle \Gamma_n \rangle]^{-1}$ yields the following result for the total normalized current

$$\vec{J}_n^{(1)} = \langle \vec{J}_n^i \rangle + \{ \delta \vec{J}_n^i + \tilde{\Gamma}_n \bullet \vec{J}_n^i \} \quad (16)$$

Reintroduction of the phase normalization factor yields the following result for what has been called the normalized first order smoothing (NFOS) approximation [9];

$$\vec{J}^{(n1)} = \{ \vec{J}_n^i + \tilde{\Gamma}_n \bullet \vec{J}_n^i \} \phi \quad (17)$$

where

$$\phi = \exp [j(k_{sz} - k_{iz})\zeta] \quad (18)$$

Taking the fluctuating part of (17) yields;

$$\delta \vec{J}^{(n1)} = \delta \vec{J}^i + \langle \phi \rangle \tilde{\Gamma}_n \bullet \vec{J}_n^i + \delta \phi \tilde{\Gamma}_n \vec{J}_n^i \quad (19)$$

Comparing (19) with (11), i.e. NFOS with FOS, clearly shows that (19) contains an additional term which does not go to zero as $k_o^2 \langle \zeta^2 \rangle \rightarrow \infty$; in particular this term is

$$\delta \phi \tilde{\Gamma}_n \bullet \vec{J}_n^i \quad (20)$$

If it turns out that the operator involving $L(\Gamma_n)$ in (15) must be retained then one can deal with the Fourier transform of (15) because the operator becomes a simple multiplicative factor in the transform domain. Dealing with the transform of (15) or

(19) has the added advantage of producing a quantity that is proportional to the scattered field. In the transform domain, (20) becomes

$$\tilde{\delta\phi} \otimes \tilde{\Gamma}_n \bullet \tilde{J}_n^i \quad (21)$$

where the symbol \otimes denotes convolution. This term has an important interpretation which will be discussed later in the paper.

Capabilities and Limitations of Normalized Smoothing

The limitation of the FOS approximation is that it will fail if the surface height becomes large relative to a wavelength. If the surface height is small, FOS has no limitations [8]. Comparisons with exact numerical solutions will be required to establish the height range over which FOS exhibits acceptable accuracy. However, this can be accomplished for one dimensionally rough surfaces where numerical solutions are available.

Establishing the capabilities and limitations of NFOS is somewhat more difficult. To accomplish this, the following approach has been adopted. The NFOS approximation is based on the neglect of the last term of (15), i.e.

$$\delta [L\delta\Gamma_n \bullet \delta\tilde{J}_n] \quad (22)$$

The only time this term is known to be of negligible consequence is when the surface height is small because this is when NFOS becomes FOS. Thus, the question to be addressed is when does the term in (22) look like it corresponds to the case of small height? More specifically, the question should be when does the propagator $\delta\Gamma_n$ look the same as with the small height FOS approximation?

The height dependence in $\delta\Gamma_n$ appears as the difference

$$\Delta\zeta = \zeta(\vec{\rho}) - \zeta_o(\vec{\rho}_o) \quad (23)$$

both in the phase and the amplitude; see (4d), (3d), and (3e). Starting off with

$$\zeta(\vec{\rho}) = \zeta_s(\vec{\rho}) + Z \quad (24)$$

where $\zeta_s(\vec{\rho})$ is a spatially varying small height component and Z is a random number yields

$$\zeta(\vec{\rho}) - \zeta_o(\vec{\rho}_o) = \zeta_s(\vec{\rho}) - \zeta_s(\vec{\rho}_o) \quad (25)$$

which is the difference of two small surface heights and just like what appears in FOS. The decomposition in (24) corresponds to the sum of a spatially variable small height, $\zeta_s(\vec{\rho})$, superimposed on a randomly elevated planar surface, Z . Thus, for this class of surface roughness, NFOS will be very accurate. Equation (24) suggests that the next class of surface to be considered is a small variable roughness superimposed on a randomly elevated and tilted planar subsurface, i.e.

$$\zeta(\vec{\rho}) = \zeta_s(\vec{\rho}) + Z + \vec{m} \cdot \vec{\rho} \quad (26)$$

where

$$\vec{m} = m_x \hat{x} + m_y \hat{y}$$

and m_x and m_y are the x and y components of slope of the planar subsurface. Substituting (26) into (23) yields

$$\zeta(\vec{\rho}) - \zeta_o(\vec{\rho}_o) = \zeta_s(\vec{\rho}) - \zeta_s(\vec{\rho}_o) + \vec{m} \cdot (\vec{\rho} - \vec{\rho}_o) \quad (27)$$

The propagator $\delta\Gamma_n$ in (22) for the height in (27) is exactly like a small height situation only with a tilt to the underlying planar surface. However, the term in the small height case corresponding to (22) is insensitive to global tilting of the underlying surface. This implies that (22) is negligible for the case of small scale roughness superimposed on a randomly elevated and tilted planar subsurface. It should be noted that there is no restriction on the magnitude of the height or the slope of the subsurface.

The next step in constructing a surface to which NFOS will apply is to add some underlying curvature to (26), i.e.

$$\zeta(\vec{\rho}) = \zeta_s(\vec{\rho}) + Z + \vec{m} \cdot \vec{\rho} + \vec{c} \cdot \vec{R} \quad (28)$$

where \vec{c} is a vector having (constant) curvature components, i.e.

$$\vec{c} = c_{xx} \hat{x} + c_{yy} \hat{y} + c_{xy} \hat{z}$$

and

$$\vec{R} = (x^2/2) \hat{x} + (y^2/2) \hat{y} + (xy) \hat{z}$$

The height difference at two points, using (27), will be

$$\zeta(\vec{\rho}) - \zeta_o(\vec{\rho}_o) = \zeta_s(\vec{\rho}) - \zeta_s(\vec{\rho}_o) + \vec{m} \cdot (\vec{\rho} - \vec{\rho}_o) + \vec{c} \cdot (\vec{R} - \vec{R}_o) \quad (28)$$

Unfortunately, the presence of the curvature terms make it impossible to make the NFOS propagator $\delta\Gamma_n$ look like the FOS propagator $\delta\Gamma$. Hence, when the underlying surface has a nonnegligible curvature, it is no longer possible to rigorously argue that NFOS is accurate. The inability of NFOS to properly account for a nonnegligible underlying surface curvature comprises the fundamental limitation of NFOS. However, it should be noted that NFOS provides a result which is very much like the conventional composite surface scattering model. This can be understood by examining (19). The first term on the right hand side is the Kirchhoff approximation which dominates about the specular direction. The second term is negligible when the underlying height is large because $\langle\phi\rangle = 0$. The third term in (19) corresponds to the tilting of the Rice or Bragg scatter solution by the random slopes of the underlying surface. Equation (19) is an improvement over the composite surface model in that the small scale surface need not have small height derivatives. Conversely, it is deficient in that it permits essentially no curvature in the underlying surface. Thus, it can be said that NFOS provides a composite surface scattering model with improved accounting for the small scale surface structure but no large scale shadowing effects. Note that the primary effect of the large scale or underlying surface structure is to give rise to shadowing in the conventional composite surface model.

One possible way to improve on (19) is to iterate (15) one or more additional times. However, it is not clear what this will produce. In fact, it is well known that

accounting for such effects as shadowing may require many iterations of (15). Hence, further iteration of (15) does not appear to be a profitable effort. In the next section, a somewhat different approach will be developed to provide an improvement to the NFOS approximation.

A Combination of The Normalized Smoothing and Composite Surface Ideas

In this section, ideas suggested by both the NFOS approximation and the composite surface model will be combined to produce a more robust scattering model. The first step is suggested by the composite model and entails splitting or partitioning the surface roughness spectrum into contiguous parts denoted as large and small scale subspectra as shown in Figure 2. The partitioning wavenumber k_p is chosen to satisfy two criteria. First, it must be less than the electromagnetic wavenumber k_o ; the smaller it is relative to k_o the better. The reason for this is to insure that all large scale surface height spatial frequencies are small compared to k_o . This condition will permit the use of quasi-optical approximations in calculating the scattering from the large scale part of the surface. It is implicitly assumed that the small scale spectrum contributes most significantly to the total surface curvature, rate of change of curvature, etc. It is acceptable for the large scale spectrum to contain large surface slopes, but the higher order surface height derivatives in this spectral region must be small. The second criterion to be satisfied by the choice of k_p is that the FOS approximation, as given by (11) with $\zeta = \zeta_s$, is an adequate representation for the scattering from the small scale structure. This implies that $k_o \langle \zeta_s^2 \rangle^{1/2}$ is not too large; the exact range is yet to be determined.

The conditions imposed on the selection of k_p are not unlike those established for the composite model [12]. If anything, they are probably less demanding and it will be subsequently shown how they may be relaxed even further. In any case, it is important to note that many natural surfaces do exist for which the spectral partitioning may be accommodated.

The next step is suggested by the NFOS analysis; equation (2) is normalized by the phase factor

$$\exp[j(k_{sz} - k_{iz})\zeta_l] \quad (29)$$

which depends only on the large scale height. Equation (2) becomes

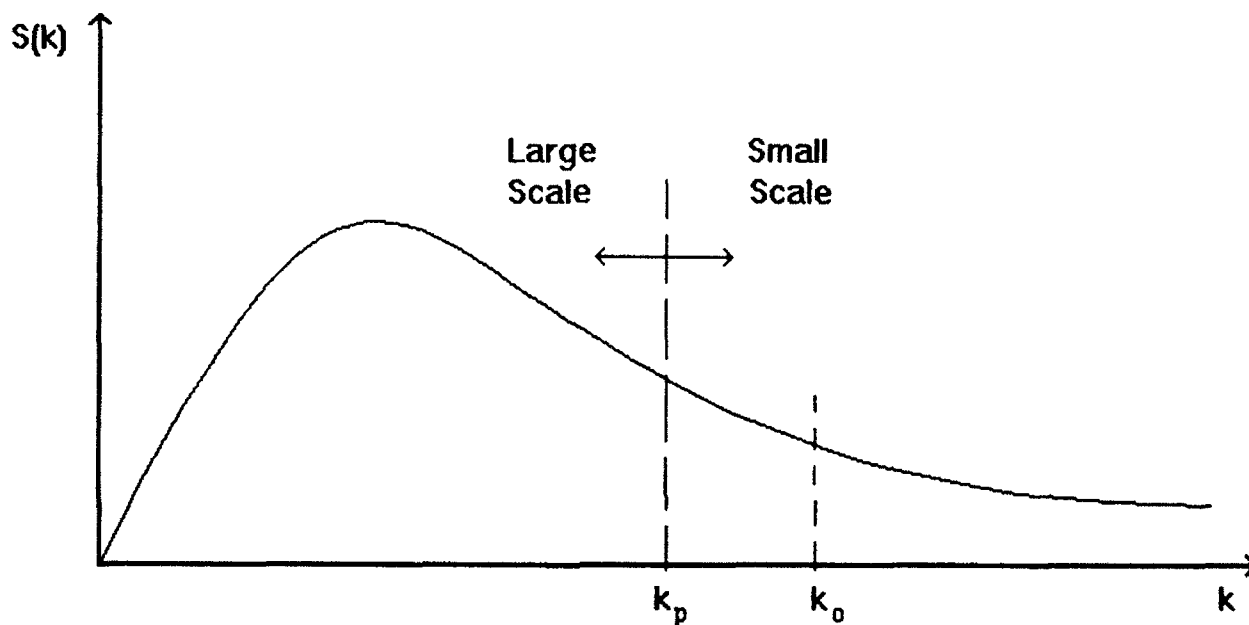


Figure 2. A partitioning of the surface height spectrum into large and small scale parts. Note that the electromagnetic wavenumber, k_o , should be larger than the partitioning wavenumber, k_p , in order to keep the large scale spectrum smoothly varying relative to λ_o .

$$\vec{J}_{nl} = \vec{J}_{nl}^i + L \Gamma_{nl} \bullet \vec{J}_{nl} \quad (30)$$

where

$$\vec{J}_{nl} = \vec{J} \exp[-j(k_{sz} - k_{iz})\zeta_l] \quad (31a)$$

$$\vec{J}_{nl}^i = 2\vec{N} \times \vec{H}^i(\vec{r}) \exp[jk_{iz}\zeta_l + jk_{sz}\zeta_s] \quad (31b)$$

$$\Gamma_{nl} = -2 \frac{\partial g}{\partial |\Delta \vec{r}|} \exp[jk_{sz}\Delta\zeta_s + jk_{iz}\Delta\zeta_l] \vec{N} \times [\hat{\Delta} \vec{r} \times \quad (31c)$$

and the argument of the free space Green's function, as a reminder, is

$$[\Delta x^2 + \Delta y^2 + \Delta \zeta^2]^{1/2} = [\Delta x^2 + \Delta y^2 + (\Delta \zeta_s + \Delta \zeta_l)^2]^{1/2} \quad (32)$$

When dealing with the NFOS approximation, it was found that NFOS would work very nicely for an underlying surface comprising a randomly elevated and inclined plane. The way to adapt this result to (30) is as follows. The integral in (30) is split into two contiguous parts; the first part encompasses some yet to be determined neighborhood of the point $\vec{\rho}_o = \vec{\rho}$ while the second part includes the remainder of the plane, e.g.

$$L = \int \int_{-\infty}^{\infty} (\bullet) d\vec{\rho}_o = \int \int_{|\vec{\rho} - \vec{\rho}_o| \leq Q} (\bullet) d\vec{\rho}_o + \int \int_{|\vec{\rho} - \vec{\rho}_o| > Q} (\bullet) d\vec{\rho}_o \quad (33)$$

Expanding ζ_{lo} about the point $\vec{\rho}_o = \vec{\rho}$, Q is determined by requiring that the two term approximation for ζ_{lo} , i.e.

$$\zeta_{lo} \approx \zeta_l - \nabla \zeta_{lo}|_{\vec{\rho}_o=\vec{\rho}} \bullet \Delta \vec{\rho} \quad (34)$$

is accurate when

$$|\Delta \vec{\rho}| = |\vec{\rho} - \vec{\rho}_o| \leq Q \quad (35)$$

Substituting (34) in (32) yields, for (35) to be satisfied,

$$|\Delta \vec{r}| = [\Delta x^2 + \Delta y^2 + (\Delta \zeta_s + \zeta_{lx} \Delta x + \zeta_{ly} \Delta y)^2]^{1/2} \quad (36)$$

Substituting this result in the propagator in (31c) shows that as long as (35) is satisfied, the integral term looks like the case of small scale roughness superimposed on a randomly elevated and tilted plane. Hence, the integral equation in (30) can be written as

$$\vec{J}_{nl} = \vec{J}_B + \frac{L}{O} \Gamma_{nl} \bullet \vec{J}_{nl} \quad (37)$$

where

$$\vec{J}_B = \vec{J}_{nl}^i + \frac{L}{I} \Gamma_{nl} \bullet \vec{J}_{nl} \quad (38)$$

and

$$\frac{L}{I} = \iint_{|\vec{\rho} - \vec{\rho}_o| > Q} (\bullet) d\vec{\rho}_o \quad (39)$$

$$\frac{L}{O} = \iint_{|\vec{\rho} - \vec{\rho}_o| > Q} (\bullet) d\vec{\rho}_o \quad (40)$$

In view of the restricted range of the integral in (38) and the fact that $\nabla \zeta_l$ is constant in this range, it seems obvious that \vec{J}_B is the current \vec{J}_{nl} with the large scale slope held constant,

$$\vec{J}_B = \vec{J}_{nl} | \nabla \zeta_l \quad (41)$$

However, rather than approaching the solution from this somewhat heuristic point of view, consider the following approach. According to (31b), \vec{J}_{nl}^i is independent of ζ_l . In addition, by virtue of the way the area I in (38) was constructed, Γ_{nl} is also independent of ζ_l in (38). Hence, \vec{J}_B will be independent of the large scale surface height ζ_l . This observation implies that (37) can be solved by iteration in which each integration is performed by the stationary phase approximation provided $k_{iz}^2 \langle \zeta_l^2 \rangle \gg 1$. Stationary phase can be used because of the phase factor

$$\exp(jk_{iz}\Delta\zeta_l)$$

in Γ_{nl} (see (31c)) and because the point

$$\vec{\rho}_o = \vec{\rho}$$

has been specifically excluded from the range of integration in (37). The solution of (37), under these conditions, can be inferred directly from previous work on integral equations of this kind [13];

$$\vec{J}_{nl} = S_i^I \vec{J}_B + \text{roms} \quad (42)$$

where S_i^I is the large scale dependent, incident shadowing function defined to be unity if a point on the surface is not shadowed from the incident field by the large scale surface and zero if it is. It is important to note that only the large scale surface enters into the determination of the shadowing function. The term "roms" in (42) stands for "ray optic multiple scattering" and it represents the rays that bounce around on the surface (by two or more bounces) before traveling into the upper free space. Although these rays can be accounted for, it is a most tedious "book keeping" effort to do so and they will be ignored in the remainder of this paper, e.g.

$$\vec{J}_{nl} \approx S_i^I \vec{J}_B \quad (43)$$

Substituting (43) in (37) yields

$$\vec{J}_B = \vec{J}_{nl}^i + \frac{1}{I} S_i^I \Gamma_{nl} \bullet \vec{J}_B \quad (44)$$

It is interesting to consider what (43) and (44) imply relative to shadowing. If the area I is shadowed then S_i^I is zero in (44) and this yields

$$\vec{J}_B = \vec{J}_{nl}^i$$

However, substituting this result in (43) produces

$$\vec{J}_{nl} = 0 \quad (45)$$

Conversely, if the area I is not shadowed so that $S_i^I = 1$ then using (43) in (44) yields

$$\vec{J}_{nl} = \vec{J}_{nl}^i + L_I \Gamma_{nl} \bullet \vec{J}_{nl} \quad (46)$$

Thus, (45) and (46) are the appropriate solutions when either the point $(\vec{\rho}, \zeta_l + \zeta_s)$ is shadowed or illuminated, respectively, by the incident field. Consequently, it is only necessary to multiply the solution of (46) by the large scale shadowing function to have a complete accounting for the effects of shadowing. Equation (46) can be solved using the method of smoothing because it was set up to be amenable to such a method. The only point of caution is to recall that the large scale surface slope is taken to be constant in the area I . Proceeding as in obtaining (16) yields ¹

$$\vec{J}_{nl}^{(1)} = \vec{J}_{nl}^i + L_I \Gamma_{nl} \bullet \langle \vec{J}_{nl}^i \rangle \quad (47)$$

or, with the inclusion of the shadowing function,

$$\vec{J}_{nl} \approx S_i^I \{ \vec{J}_{nl}^i + L_I \Gamma_{nl} \bullet \langle \vec{J}_{nl}^i \rangle \} \quad (48)$$

where, in review,

$$\vec{J}_{nl}^i = 2H_o \vec{N} \times \hat{h}_i \exp[j(k_{sz} - k_{iz})\zeta_s - j\vec{k}_{it} \bullet \vec{\rho}] \quad (49a)$$

$$\langle \vec{J}_{nl}^i \rangle = 2H_o \langle \vec{N} | \nabla \zeta_l \times \hat{h}_i \langle \exp[j(k_{sz} - k_{iz})\zeta_s] \rangle \exp(-j\vec{k}_{it} \bullet \vec{\rho}) \quad (49b)$$

$$\Gamma_{nl} = -2 \frac{\partial g}{\partial |\Delta \vec{r}|} \exp[jk_{sz}\Delta\zeta_s + jk_{iz} \nabla \zeta_l \bullet \Delta \vec{\rho}] \vec{N} \times [\hat{\Delta} \vec{r} \times \quad (49c)$$

¹It is assumed that the Fourier transform of the average propagator $\langle \tilde{\Gamma}_{nl} \rangle$ is negligibly small. If this is the case then $L_I \langle \Gamma_{nl} \rangle$ may also be ignored. If the average propagator cannot be ignored, it can be shown that (47) becomes

$$\vec{J}_{nl}^{(1)} = [1 - L_I \langle \Gamma_{nl} \rangle]^{-1} \{ \vec{J}_{nl}^i + L_I \delta \Gamma_{nl} [1 - L_I \langle \Gamma_{nl} \rangle]^{-1} \langle \vec{J}_{nl}^i \rangle \}$$

$$\Delta\zeta_s = \zeta_s - \zeta_{so} \quad (49d)$$

$$\Delta\vec{\rho} = (x - x_o)\hat{x} + (y - y_o)\hat{y} = \Delta x\hat{x} + \Delta y\hat{y} \quad (49e)$$

$$\nabla\zeta_l = \partial\zeta_l/\partial x \hat{x} + \partial\zeta_l/\partial y \hat{y} \quad (49f)$$

$$\Delta\vec{r} = \{\Delta x^2 + \Delta y^2 + (\Delta\zeta_s + \nabla\zeta_l \cdot \Delta\vec{\rho})^2\}^{1/2} \quad (49g)$$

Also, H_o is the amplitude of the incident magnetic field, \hat{h}_i is its direction, and $\langle \vec{N} | \nabla\zeta_l \rangle$ is the conditional average of \vec{N} conditioned on holding the large scale slopes constant. It must also be remembered that in (48), the large scale slopes $\nabla\zeta_l$ are to be held constant. To complete the evaluation of the current, the normalizing phase factor must be reintroduced, e.g.

$$\vec{J} = S_i^l \phi \{ \vec{J}_{nl}^i + \vec{L}_I \Gamma_{nl} \cdot \langle \vec{J}_{nl}^i \rangle \} \quad (50)$$

where

$$\phi = \exp[j(k_{sz} - k_{iz})\zeta_l] \quad (51)$$

The far zone electric field scattered by the surface is given by

$$\vec{E}_s(\vec{R}) = -jk_o\eta_o g(R) \{ \tilde{\vec{J}}(\vec{k}_{st}) - [\hat{k}_s \cdot \tilde{\vec{J}}(\vec{k}_{st})]\hat{k}_s \} \quad (52)$$

where R is the distance from the origin of the coordinate system on the mean surface to the point of observation, \hat{k}_s is a unit vector pointing in this direction,

$$\vec{R} = R\hat{k}_s$$

and

$$\tilde{\vec{J}}(\vec{k}_{st}) = \int \vec{J}(\vec{\rho}, \zeta) \exp(j \vec{k}_{st} \cdot \vec{\rho}) d\vec{\rho} \quad (53)$$

where

$$\vec{k}_{st} = k_o(\sin\theta_s \cos\phi_s \hat{x} + \sin\theta_s \sin\phi_s \hat{y})$$

$$\vec{\rho} = x\hat{x} + y\hat{y}$$

Discussion of Results

Equation (50) is the central result of this paper. To the author's knowledge, it represents the first tractable description of the current induced on a multiscale surface. There are certain advantages to dealing with the current rather than the far zone scattered field. This becomes especially clear if it is desirable to study Fresnel zone scattering from a rough surface.

While (50) contains a number of approximations, it represents a considerable improvement over the conventional composite surface scattering model. Paramount among these is the fact that the small scale part of the surface need not have small surface height derivatives. This result follows from the use of FOS to estimate the current induced on the surface within the surface area I . In the conventional composite model, boundary perturbation theory is used to estimate the effects of the small scale structure and it is much more limited in its range of applicability. Another subtle improvement provided by (50) is use of a finite area, I , rather than an infinite one. This finite area is tailored to the large scale slopes. That is, it is the largest area over which the effects of large scale curvature may be ignored. Thus, if one were dealing with surfaces for which the large scale rms slopes are not too large but large scale rms curvature is moderate² then I in (50) would have to be decreased relative to its value when the rms slope-to-curvature ratio were larger. It should be noted that this effect builds into the solution a dependence upon the large scale curvature that is quite independent of the method used to analyze the scattering from the large scale surface structure. This dependence requires further study in order to understand its full importance. However, it is possible to provide some estimate of the effect of reducing I as follows. If I is infinite,

$$\lim_{I \rightarrow \infty} \Gamma_{nl} \cdot \langle \vec{J}_{nl}^i \rangle \rightarrow \tilde{\Gamma}_{nl}(\vec{\rho}, -\vec{k}_{it}) \cdot \vec{j}_{nl}^i \quad (54)$$

²It is assumed that the large scale curvature is sufficiently small to permit the use of stationary phase integration techniques on the large scale surface.

where the tilde denotes the Fourier transform (from $\vec{\rho}_o$ to $-\vec{k}_{it}$) and \vec{j}_{nl}^i is a constant vector. If I is finite, it is necessary to convolve (54) with a transformed support function representing the finite size of I . This convolution has the effect of blurring or smearing the sharply defined effects contained in $\tilde{\Gamma}_{nl}$.

It is interesting to see how (50) reduces to the conventional composite surface scattering model. The details are very involved and will only be highlighted here. The term

$$S_i^I \phi \langle \vec{j}_{nl}^i \rangle$$

in (50) is what gives rise to the quasi-optical part of the scattering that is dominant in and about the specular direction. Note that this term contains large scale incidence shadowing (S_i^I), deep phase modulation associated with the large scale height (see equation (51)), and attenuation due to the small scale height ($\langle \vec{j}_{nl}^i \rangle$). Apart from this small scale dependent attenuation, the remaining factors are large scale dependent and have been treated previously by Sancer [14]. If the small scale surface height is very small and the large scale curvature is so small that

$$\frac{L}{I} \rightarrow L = \int \int_{-\infty}^{\infty} (\bullet) d\vec{\rho}_o$$

then the terms

$$\delta \vec{j}_{nl}^i + \frac{L}{I} \Gamma_{nl} \bullet \langle \vec{j}_{nl}^i \rangle$$

in (50) will go to the Rice boundary perturbation solution on a tilted plane; the tilt being provided by the large scale surface slopes. After multiplying by $S_i^I \phi$, taking the Fourier transform to generate the scattered field, squaring and averaging to produce the second moment of the scattered field, the net effect is to obtain a smeared Bragg scattering. The smearing results from the range of possible values for the large scale slopes as dictated by the probability density function of the slopes. Thus, (50) reproduces the conventional composite surface model under the same set of assumptions that are usually associated with the conventional model.

There is one final point about the method introduced here that warrants

discussion. While it is clear that a small scale surface can always be split off from the total surface, it is not so obvious that the resulting large scale features will satisfy the criteria placed on them. For example, it may be necessary to include wave diffraction effects in the solution of (37) rather than just wave reflection and shadowing as in (43). In a purely formal sense, the solution of (37) is given by

$$\vec{J}_{nl} = [1 - \frac{L}{O} \Gamma_{nl}]^{-1} \bullet \vec{J}_B \quad (55)$$

so the integral equation for \vec{J}_B is

$$\vec{J}_B = \vec{J}_{nl}^i + \frac{L}{I} \Gamma_{nl} \bullet [1 - \frac{L}{O} \Gamma_{nl}]^{-1} \vec{J}_B \quad (56)$$

and within the confines of first order smoothing

$$\delta \vec{J}_B^{(1)} = [1 - \langle A \rangle]^{-1} \bullet \{ \delta \vec{J}_{nl}^i + \frac{L}{I} \delta A \bullet [1 - \langle A \rangle]^{-1} \bullet \langle \vec{J}_{nl}^i \rangle \} \quad (57)$$

where

$$A = \Gamma_{nl} \bullet [1 - \frac{L}{O} \Gamma_{nl}]^{-1}$$

Thus, (55) becomes

$$\vec{J}_{nl}^{(1)} = [1 - \frac{L}{O} \Gamma_{nl}]^{-1} \bullet \{ [1 - \langle A \rangle]^{-1} \bullet \langle \vec{J}_{nl}^i \rangle + \delta \vec{J}_B^{(1)} \} \quad (58)$$

Equation (58) represents an approximate solution for the normalized current based on an exact solution for the long range interactions on the surface and a first order smoothing approximation for the short range interactions. Equation (55) represents the former while (57) represents the latter. It is clear from (55) and (58) that the long range solution is highly dependent upon ones ability to invert the operator $[1 - \frac{L}{O} \Gamma_{nl}]$. In the optical limit, (43) shows that

$$[1 - \frac{L}{O} \Gamma_{nl}]^{-1} \approx S_i^t \quad (59)$$

where ray optic multiple scattering terms have been ignored. If wave diffraction is to

be included, it will be necessary to do something like suggested by Chaloupka and Meckelburg [15] and Ansorge [16]. The important point is that the new method presented in this paper not only improves on the classical composite surface scattering model but it clearly shows how further improvements must proceed.

Summary

This paper adapts and combines the fundamental principles comprising the conventional composite surface scattering model and the normalized first order smoothing (NFOS) approximation to provide a new approach to the problem of scattering by multiscale surfaces. The conventional composite surface model suggests partitioning the surface into two subscales which have rms heights that are small and large relative to a wavelength. The NFOS approximation suggests normalizing the basic integral for the current induced on the surface by the large scale height dependent phase factor contained in the Kirchhoff term to produce an integral equation that is more amenable to the first order smoothing approximation.

The real heart of the improvement rendered by this approach stems from splitting the integral in the current integral equation into the sum of one encompassing the immediate neighborhood of the observation point and one including all of the remaining surface. The extent of the former integration is chosen such that the large scale surface has essentially a constant slope within this region. The sum of this integral and the Kirchhoff term act as the Born term in the integral equation involving the remaining integral term. This equation may be solved by iteration in the high frequency limit and, hence, yields the shadowed Born term as the solution. Thus, the net effect of the large scale solution at this point is simply to shadow the new Born term. The solution of the resulting finite range integral equation is accomplished by first order smoothing and is valid as long as the small scale height is not too large (relative to the wavelength) but its validity is independent of the surface slopes, curvature, etc.

The net result of this analysis is an expression for the surface current which contains all of the fundamental features of the conventional composite surface scattering model plus some marked improvements. The latter entail accounting for more general small scale surface structure and an estimate of the effects of the large scale surface curvature on the scattering from the small scale features. Finally, this result is amenable to correction for such phenomenon as high frequency diffraction by the large

scale surface. Although it is essential to expose this result to extensive computation for the purpose of establishing its range of validity, the basic merits of the new result are clear. Of particular note is the fact we now have an expression for the current induced on a multiscale rough surface rather than just one of scattered field moments. The advantages of this knowledge are obvious.

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